PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Scientific Astronomy in Antiquity

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Scientific astronomy in antiquity

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[Plates 1 and 2]

The character and content of Babylonian scientific or mathematical astronomy, as we know it from texts of the last half millennium B.C., are sketched. This late-Babylonian astronomy is set in contrast to earlier Babylonian astronomy as well as to the kinds of astronomy found in other ancient cultures, and an attempt is made at a very broad classification of such pre-scientific astronomies.

The lateness and uniqueness of Babylonian mathematical astronomy is emphasized, and it is shown that its creation depended upon the availability of a peculiar set of ingredients, e.g., a particular type of mathematics, and a tradition of making and recording observations of certain astronomical phenomena.

It is finally argued that all subsequent varieties of scientific astronomy, in the Hellenistic world, in India, in Islam, and in the West – if not indeed all subsequent endeavour in the exact sciences – depend upon Babylonian astronomy in decisive and fundamental ways.

In the present paper I shall describe to you, of necessity in barest and crudest outline, the character and content of Babylonian mathematical astronomy which was the first and highly successful attempt at giving a refined mathematical description of astronomical phenomena. Further, I shall try to give expression to my conviction that subsequent scientific astronomy – in Hellenistic Greece, in India, in Islam, and in the Latin West – depended in decisive ways upon Babylonian astronomy.

My choice of a title for this lecture was, I fear, somewhat clumsy – mathematical or theoretical astronomy in antiquity might better describe my topic – and so to avoid misunderstanding, I should like to make clear what I mean by scientific astronomy; and let me begin by giving some examples of what I do not wish to include in that category. To talk about what I shall not talk about may not be as idle as it sounds, for it will serve to set my principal topic in relief, and also to provide a first swift glimpse of subjects which will be dealt with more properly by others. I shall exclude astronomical observations from my discussion; not that I consider them irrelevant to my topic, on the contrary, but a treatment of observational techniques and records would lead me too far from my main concern. I shall, none the less, have something to say later about the connexion between Babylonian observations and theories.

I find it useful to distinguish between two levels of prescientific astronomy. To the less advanced level I assign astronomical achievements such as the naming of prominent stars and constellations; drawing the distinction between fixed stars and planets; the awareness of the morning star and the evening star being just different appearances of one and the same celestial body, namely Venus; the realization that a fixed star which is not circumpolar always rises and sets at the same two places on the horizon, while the Sun, Moon, and planets do not; the discovery that the first appearance of a fixed star after its interval of invisibility happens at the same time of year and may be used as a seasonal indicator.

As examples of this last practice I need only remind you of the part of Hesiod's *Works and days*, beginning with line 383, in which certain rustic tasks are tied to the first or last visibility of certain fixed stars, and of the ancient Egyptian use of Sirius (or Sothis) at its first visibility as a herald of the time of flooding of the Nile. This sort of primitive astronomical knowledge, a farmer's or shepherd's astronomy, if you will, I should not be at all surprised at finding in any settled community, whether literate or not.

The more advanced level of prescientific astronomy you will find in some, but not all, of the ancient literate cultures, namely, in China, in the pre-Hellenistic and early Hellenistic Greek world, in early Mesopotamia, and in Mesoamerica. You will hear about these things in some detail from others; here let me say that however different such early astronomies may be in their aims and approach, they have as a common characteristic that they employ various cycles concerning the motion of the Sun, Moon and the five classical planets, that is, those visible to the naked eye: Mercury, Venus, Mars, Jupiter and Saturn.

Let me give some examples, and as the first the so-called 'Metonic' cycle which sets 19 years equal to 235 synodic months. This cycle which happens to be very excellent was the basis of the calendar introduced by Meton in Athens. We find it in Babylonia and China as well.

My second example is the Venus cycle of 8 years during which Venus runs its synodic course five times; that is, it becomes a morning star five times, an evening star five times, retrograde five times, and so on, and at the end of which it returns very nearly to the same place with respect to the stars and with respect to the Sun. This cycle was known in Babylonia and remained in use, because of its excellence, with but slight modifications, until the end of cuneiform literature, side by side with much more sophisticated schemes for the other planets. It was also known in China and it plays an important role in Mayan astronomy.

Thirdly, there are various so-called eclipse cycles. Though too crude to yield reliable predictions of eclipses, such cycles could be used, as we now know, for issuing eclipse warnings in advance, that is, for pointing to syzygies (i.e. conjunctions or oppositions of Sun and Moon) at which there was a danger of either a solar or a lunar eclipse, while guaranteeing safety or freedom from eclipses elsewhere (Aaboe 1972). One such cycle consists of 135 lunar months in the course of which one issues 23 eclipse warnings according to a certain pattern. We have evidence of this cycle from Babylon and it was known and used in China in the Han period; the Mayas used a cycle of three times 135 months.†

It is really not surprising that such cycles are discovered in literate civilizations that have made a habit of recording important events, among them celestial phenomena. Thus I would say that the occurrence of the same period of this character in two civilizations is not by itself evidence of contact between them. Indeed, it would be absurd to suggest that a connexion existed between the Babylonian and the Mayan civilizations just because the Venus cycle and a certain eclipse cycle are known in both. (It is perhaps foolish to make even such a negative statement, for it might lead some intrepid enthusiast to sail on inflated goat skins from the Euphrates to the Panama canal.)

I shall here mention three uses of such periods. The first is for calendric purposes, where various cycles are combined to form larger cycles which assure repetition of several kinds of phenomena. The Metonic cycle which I already mentioned is a very simple example. In China and in Mesoamerica much more complicated periods are built up, as you will doubtless hear.

Secondly, such periods may be used to furnish parameters for astronomical models of various kinds, as I shall mention later in my discussion of some Greek geometrical models.

† The factor 3 is easily explained because 135 months happen to be two-thirds of a day more than a whole number of days, so that thrice this period is very nearly a whole number of days as required by Mayan astronomy in which one day, and not one lunation, was the basic unit.

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Thirdly, they can be made to yield predictions of astronomical events, particularly when used in conjunction with a series of observational records. A perfect example of this approach is the class of late Babylonian texts which Sachs has called Goal Year Texts (Sachs 1948). A goal year text written for this year, A.D. 1972, would contain information about the planets and the Moon in the following fashion: under the heading of Jupiter it would tell about Jupiter in 1901, for what Jupiter did 71 years ago it will do again this year on the same dates, 71 years being a very good period of Jupiter; for Venus it would tell what happened in 1964, for 8 years is a good Venus period; for Mercury it would have what happened 46 years ago; for Saturn what happened 59 years ago, and so on for Mars and the Moon. The information presented would be taken from observational records for the relevant years. This turns out to be a very efficient way of giving astronomical predictions, but it still falls short of what I mean by a truly scientific astronomy.

I do not wish to call an astronomical theory scientific until it gives us control over the irregularities within each period and thus frees us from constant consultation of observational records.†

What I mean by a scientific astronomical theory is then a mathematical description of celestial phenomena capable of yielding numerical predictions that can be tested against observations. I think this distinction will become clear when I introduce you to a truly scientific astronomical text.

My first illustration (figure 1, plate 1) shows a photograph of the reverse of a Babylonian clay tablet, pieced together from many fragments, which was published most recently by Neugebauer (1955) as A.C.T. no. 122. The scribe tells us in the colophon that it was written in year 209 of the Seleucid Era, month IX, day 18, and it gives us information about new moons for the three years 208, 209 and 210 of the same era, so that the text must, at least in part, be a forecast. I chose this text not because it is typical of the material we have to work with, which, alas, it is not, but rather because it is one of the most complete examples of a lunar text that has reached us.

The tablet has the oblong shape characteristic of lunar ephemerides, as we are accustomed to call these texts, even though they do not proceed day by day, but month by month.

Figure 2*a*, *b*, *c* shows Neugebauer's transcription and restoration of the text, and one's first impression, I dare say, is of quantities and quantities of numbers arranged very neatly in columns that continue from the obverse over the bottom edge onto the reverse. Horizontal alinement is strictly observed in this text, as in all similar texts, and indeed all the numbers in one line running across from the left edge to the right concern one particular month or rather, in this case, the situation around the new moon, that happens at the end of one month, and the first visibility of which signals the beginning of the next. The first column, which is not preserved, but which can be reconstructed with confidence, gives the year number and the month.

The first line of the text concerns month XII of the year 207 of the Seleucid Era, and whatever is told there has to do with the conjunction that took place at the end of March in 104 B.C., to transform to a date in our era (Parker & Dubberstein 1956). As you read down the first column, we simply have years and the corresponding months listed in order. You will

† A modern parallel to the Goal Year texts is the kind of meteorology where a replica of the current pattern of temperatures, pressures, and wind velocities is found in past records, and predictions are given according to what happened then; this methodology would, in my usage, be prescientific.

verse [O]		I	II		III	IV	V		VI	[VII]
	XII	[29, 8, 3]9, 18	2, 2, 6, 20	[ḫ]un	2, 56	1, 32	6, 5, 30	sig	[11, 30]	3, 59, 52, 30
3, 28	I	[28, 50, 39,] 18	[5]2, 45, 38	múl	3, 14	1, 23	9, 46, 30	sig	[11, 16, 10]	4, 22, 22, 30
	II	[28, 3]2, 39, 18	29, 25, 24, 56	múl	3, 26	1, 17	5, 54	sig	11, [52, 10]	4, 14, 1, 40
	III	[28,] 14, 39, 18	27, 40, 4, 14	maš	3, 34	1, 13	2, 1, 30	sig	12, 2[8, 10]	3, 51, 31, 40
	IV	[2]8, 24, 40, 2	26, 4, 44, 16	kušú	3, 32	1, 14	1, 51	bar	13, 4, 10	3, 29, 1, 40
	V	[2]8, 42, 40, 2	24, 47, 24, 18	a	3, 24	1, 18	2, 43, 30	nim	[13,] 40, 10	3, 6, 31, 40
	VI	29, ., 40, 2	23, 48, 4, 20	absin	3, 9	1, 25	6, 36	nim	14, 16, 10	2, 44, 1, 40
	VI_2	29, 18, 40, 2	23, 6, 44, 22	rín	2, 51	1, 34	9, 16	nim	14, 52, 10	2, 21, 31, 40
	VII	[2]9, 36, 40, 2	22, 43, 24, 24	gír-tab	2, 36	1, 42	5, 23, 30	nim	15, 4	1, 59, 1, 40
	VIII	29, 54, 40, 2	[22, 38,] 4, 26	pa	2, 27	1, 46	1, 31	nim	14, 28	2, 8, 37, 30
	IX	[29,] 51, 17, 5[8]	[22, 29, 22,] 24	máš	2, 27	1, 46	2, 21, 30	bar	13, 5[2]	2, 31, 7, 30
	X	[29,] 33, 17, 58	[22, 2, 40, 22	g]u	2, [3]6	1, 42	[3, 14	sig]	[13, 16]	2, 53, 37, 30
	XI	[2]9, 15, 17, 58	21, 17, 58, 20	zib-me	2, [50]	1, [35]	[7, 6, 30	sig]	[12, 40]	3, 16, 7, 30
	XII	[2]8, 57, 17, 58	20, 15, 16, 18	ḫun	3, 8	1, 26	[8, 45,] 30	sig	12, [4]	3, 38, 37, 30
3, 29	I	[2]8, 39, 17, 58	18, 54, 34, 16	múl	3, 22	1, 19	[4,] 53	sig	11, 28	4, 1, 7, 30
	II	[28,] 21, 17, 58	17, 15, 52, 14	maš	3, 32	1, 14	[1,] ., 30	sig	11, 18, 10	4, 23, 37, 30
	III	[28, 1]8, 1, 22	15, 33, 53, 36	kušú	3, 35	1, 12	[2,] 52	bar	11, 5[4], 10	4, 12, 46, 40
	IV	[28, 3]6, 1, 22	14, 9, 54, 58	a	3, 28	1, 16	[3,] 44, 30	nim	12, 30, 10	3, 50, 16, 40
	V	[28, 54, 1,] 22	13, 3, 56, 20	absin	3, 15	1, 22	[7, 37	ni]m	[13, 6, 10]	3, 27, 46, 40
	VI	[29, 12, 1, 2]2	12, 15, 57, 42	rín	2, 5[8]	[1, 31]	[8, 15	nim]	[13, 42, 10]	3, 5, 16, 40

erse [O]		I	II		III	IV	V		VI	VII
	VII	[29, 30,] 1, 22	11, 45, 59, 4	gír-tab	2, 40	1, 40	4, [22, 30	nim]	[14, 18, 10]	[2, 42, 46, 40]
	VIII	[29,] 48, 1, 22	11, 34, ., 26	pa	2, 29	1, 45	., [30	nim]	[14, 5]4, [10]	2, [20, 16, 40]
	IX	[2]9, 57, 56, 38	11, 31, 57, 4	máš	2, 25	1, 47	., [22,] 30	bar	15, 2	1, 57, 46, 40
	X	29, 39, 56, 38	11, 11, 53, 42	gu	2, 31	1, 44	4, [1]5	sig	14, 26	2, 9, 52, 30
	XI	[2]9, 21, 56, 38	10, 33, 50, 20	zib-me	2, 43	1, 38	8, 7, 30	sig	13, 50	2, 32, 22, 30
	XII	[2]9, 3, 56, 38	9, 37, 46, 58	ḫun	3, 1-	1, 29	7, 44, 30	sig	13, 14	2, 54, 52, 30
3, 30	I	[28,] 45, 56, 38	8, 23, 43, 36	múl	3, 18	1, 21	3, 52	sig	12, 38	3, 17, 22, 30
	II	[28, 2]7, 56, 38	6, 51, 40, 14	maš	3, 29	1, 15	., ., 30	bar	12, 2	3, 39, 52, 30
	III	[28, 1]1, 22, 42	5, 3, 2, 56	kušú	3, 35	1, 12	., 53	nim	11, 26	4, 2, 22, 30
	IV	[28, 2]9, 22, 42	3, 32, 25, 38	a	3, 31	1, 14	4, 45, 30	nim	11, 20, 10	4, 24, 52, 30
	V	[28,] 47, 22, 42	2, 19, 48, 20	absin	3, 20	1, 20	8, 38	nim	11, 56, 10	4, 11, 31, 40
	VI	[29,] 5, 22, 42	1, 25, 11, 2	rín	3, 4	1, 28	7, 14	nim	12, 32, 10	3, 49, 1, 40
	VII	[2]9, 23, 22, 42	48, 33, 44	gír-tab	2, 46	1, 37	3, 21, 30	[n]im	13, 8, 10	3, 26, 31, 40
	VIII	[2]9, 41, 22, 42	29, 56, 26	pa	2, 33	1, 43	31	bar	[1]3, 44, 10	3, [4, 1, 40]
	IX	[29,] 59, 22, 42	29, 19, 8	máš	2, 26	1, 47	1, 23, 30	sig	14, 20, 10	2, 41, 31, 40
	X	[2]9, 46, 35, 18	15, 54, 26	gu	2, 28	1, 46	5, 16	sig	14, 56, 10	2, 19, 1, 40
	XI	[29,] 28, 35, 18	29, 44, 29, 44	gu	2, 39	1, 40	9, 8, 30	sig	15	1, 56, 31, 40
	XII	[29, 10, 3]5, 18	28, 55, 5, 2	zib-me	2, 54	1, 33	6, 43, 30	sig	14, 24	2, 11, 7, 30
	XII_2	[28, 52, 35,] 18	27, 47, 40, 20	ḫun	3, 12	1, 24	2, 5[1	sig]	[13, 4]8	2, 33, 37, 30

FIGURE 2*a*

VIII	IX		X	XI				XII		
[20, 20]	[7, 19	lal]	[3, 52, 33, 30]	[še	29	1, 2, 43, 50	šú]	[29	., 30	kur]
[14, 52, 30]	[22, 11, 30	lal]	[4, ., 11]	[bar	28	5, 2, 54, 50	šú]	[29	., 26	d]u
[8, 5]	[30, 16, 30	lal]	[3, 43, 45, 10]	[gu$_4$	28	2, 46,] 40	šú	28	1, 2[9	n]im
[1, 17,] 30	31, 34	lal	3, 19, 57, 40	sig	29	., 6, 37, 40	šú	29	1, [7	kur]
[5,] 30	27, 5[2	lal]	3, 1, 9, 40	š[u]	28	3, 7, 47, 20	šú	28	1, [38	šú]
[12,] 17, 30	[15, 34, 30	lal]	[2, 50, 57, 10]	[izi	28	5, 58, 44, 30	šú]	[29	1, 16	d]u
[19, 5]	[3, 30, 3]0	tab	2, 47, 32, 10	[ki]n	28	2, 46, 16, 40	šú	[28	1, 21	ni]m
[16, 7, 30]	[19, 3]8	tab	2, 41, 9, 40	ki[n 2-kam	2]8	5, 27, 26, 20	šú	2[9	1, 1	d]u
[9, 20]	[28,] 58	tab	2, 27, 59, 40	[du$_6$]	29	1, 55, 26	šú	2[9	., 13	ni]m
[2, 32, 30]	[31,] 30, 30	tab	2, 40, 8	[api]n	28	4, 35, 34	šú	2[9	., 22	d]u
[4, 15]	[2]9, 10, 30	tab	3, ., 18	gan	29	1, 35, 52	šú	2[9	., 10	ku]r
[11, 2, 30]	18, 8	tab	3, 11, 45, 30	ab	28	4, 47, 37, 30	šú	2[9	., 29	d]u
[17, 5]0	18	tab	3, 16, 25, 30	zíz	29	2, 4, 3	šú	[29	., 29	nim]
[17,] 22, 30	17, 4, 30	lal	3, 21, 33	še	28	5, 25, 36	[šú]	[29	., 51	du]
[10, 35]	[27, 39, 30	lal]	[3,] 33, 28	b[ar]	28	[2,] 59, 4	[šú]	[28	1, 40	nim]
[3, 47, 30]	[31, 27	lal]	[3,] 52, 10, 30	gu$_4$	29	51, 14, 30	[šú]	[29	., 23	kur]
[3]	30, 29	lal	3, 42, 17, 40	sig	28	4, 33, 32, [10	šú]	[28	., 14	šú]
[9, 47, 3]0	20, 41, 30	lal	3, 29, 35, 10	šu	28	2, 3, 7, 20	[šú]	[28	., 47	nim]
[16, 3]5	4, 6, 30	lal	3, 23, 40, 10	izi	28	5, 26, [47, 30	šú]	[29	., 49	du]
[18,] 37, 30	14, 31	tab	3, 19, 47, 40	kin	29	[2, 46, 35, 10	šú]	[29	1, 15	nim]

VIII	IX		X	XI				XII		
[11, 50]	[26, 21	tab]	[3, 9, 7, 40]	[du$_6$	28	5, 55, 42, 50	šú]	[2]9	1, 35	du
[5, 2, 30]	[31, 23,] 30	tab	2, 51, 40, 10	apin	28	2, 47, 23	šú	28	1, 2	nim
1, 45	31, 47, 30	tab	2, 29, 34, 10	gan	28	5, 16, 57, 10	šú	29	1, [4	d]u
8, 32, 30	23, 15	tab	2, 33, 7, 30	ab	29	1, 50, 4, 40	šú	29	., [6]	nim
15, 20	7, 55	tab	2, 40, 17, 30	zíz	28	4, 30, 22, 10	šú	29	., 9	du
19, 52, 30	11, 57, 30	lal	2, 42, 55	še	29	1, 13, 17, 10	šú	29	15	kur
13, 5	25, 2, 30	lal	2, 52, 20	bar	28	4, 5, 37, 10	šú	28	34	šú
6, 17, 30	31, 20	lal	3, 8, 32, 30	gu$_4$	29	1, 14, 9, 40	šú	29	., 1	kur
., 30	31, 50	lal	3, 30, 32, 30	sig	28	4, 44, 42, 10	šú	28	[.,] 3	[šú]
7, 17, 30	25, 48, 30	lal	3, 59, 4	šu	28	2, 43, 46, 10	šú	28	1, 29	[nim]
14, 5	11, 43, 30	lal	3, 59, 48, 10	izi	29	43, 34, 20	šú	29	3[7	kur]
20, 52, 30	9, 9	tab	3, 58, 10, 40	kin	28	4, 41, 45	šú	29	9	[du]
14, 20	23, 29	tab	3, 50, ., 40	du$_6$	29	2, 31, 45, 40	šú	29	5[5	nim]
7, 32, 30	31, [1,] 30	tab	3, 35, 3, 10	[a]pin	29	6, 48, 50	šú	29	[1, 36	kur]
[., 45]	31, 4[6, 30	tab]	3, 13, [18,] 10	[gan	2]8	[3,] 20, 7	[šú]	[28	53	šú]
6, 2, 30	27, [7]	tab	2, 46, 8, 40	ab	29	6, 15, 40	šú	2[9	1, 4]0	kur
12, 50	[14,] 17	tab	2, 10, 48, 40	zíz	28	2, 17, 4, 20	šú	2[8]	37	nim
19, 37, 30	5, 20, 30	lal	2, 5, 47	še	28	4, 22, 51, 20	šú	28	., 4	šú
15, 35	20, 55, 30	lal	2, 12, 42	dir-še	29	35, 33, 2[0	šú]	29	49	kur

FIGURE 2*b*

obverse

XIII			XIV		XV			XVI			XVII	
[kúr	30	9, 26]	[be]	14, 15	bar	1	15, 40	26	17, 30	k[ur]	[▨▨	be]
kúr	30	11, 35	[be]	17, 40	gu$_4$	1	17, 30	26	17, 50	kur	[2]3, 4[0	be]
kúr	29	7, 57	[be]	13, 10	sig	30	13	27	17, 30	kur	18, 3[0	be]
[kúr	30	1]0, 42	[be	▨▨]	[šu	1	▨ 40]	27	16, 40	kur	16	be
[kúr	29]	7, 39	[be	12, 30	[izi	30	▨▨]	27	23, 10	kur	2[2]	be
kúr	3[0]	10, 44	be	20, 30	kin	1	[1]7[▨]	27	19, 30	kur	17	be
kúr	29	7, 48	be	15, 50	kin 2-kam	30	12	27	30, 40	kur	22, 20	be
kúr	29	4, 58	be	10, 10	du$_6$	30	8, 40	28	16, 30	kur	14	be
kúr	30	8, 23	be	17, 10	apin	1	16, 50	27	20, 30	kur	19 [▨]	be
kúr	29	5, 28	be	9, 20	gan	30	9, 10	28	14	kur	1[2 ▨	b]e
kúr	30	8, 37	be	15, 50	ab	1	16, 30	27	13, 30	kur	18 [▨	be]
kúr	29	5, 31	be	9, 30	zíz	30	10 uš	27	12	kur	2[4 ▨	be]
kúr	30	8, 21	be	14	še	1	15, 10	27	10, 30	kur	1[8 ▨	be]
[kúr	3]0	11, 8	be	18, 40	bar	1	19, 20	26	16, 30	kur	2[3 ▨	be]
[kúr	29	7, 4]2	be	11, 50	gu$_4$	30	12, 10	27	17, 10	kur	1[9 ▨	be]
[kúr	30	9,] 55	be	15, 10	sig	1	15, 20	27	17	kur	1[2 ▨	be]
[kúr	30	12,] 14	be	20, 30	šu	1	19, 20 tab	27	14, 40	kur	[▨▨	be]
[kúr	29]	8, 41	be	15	izi	30	13	27	25, 40	[kur]	[▨▨	be]
[kúr	29]	5, 11	be	8, 30	kin	30	8	28	18, 30	[kur]	[▨▨	be]
[kúr	30	7, 43]	[be	▨▨]	[du$_6$]	1	12, 20	27	2[8 ▨	kur]	[▨▨	be]

reverse

XIII			XIV		XV			XVI			XVII	
kúr	30	10, 25	be	20, 10	apin	1	17, 20	27	17 [▨	kur]	[▨▨	be]
kúr	29	7, 27	be	14, 50	gan	30	14	27	20, 10	kur	[▨▨	be]
kúr	29	4, 56	be	10, 10	ab	30	10, 20	28	9, 50	kur	[▨▨	be]
kúr	30	8, 25	be	15, 50	zíz	1	16, 20	27	11, 10	kur	[▨▨	be]
kúr	29	5, 51	be	11, 30	še	30	12	27	12	kur	2[3 ▨	be]
kúr	30	9, 17	be	16, 30	bar	1	17, 40	27	11, 20	kur	17 [▨	be]
kúr	29	6, 34	be	10, 10	gu$_4$	30	10, 50	27	20, 10	kur	2[1 ▨	be]
kúr	30	9, 30	be	15	sig	1	15, 40	27	15, 10	kur	1[6 ▨	be]
kúr	30	12, 3	be	18, 30	šu	1	18, 50	27	13, 20	kur	1[2 ▨	be]
[kúr]	29	8, 2	be	11, 50	izi	30	11, 10	28	11	kur	[▨▨	be]
[kúr]	30	9, 57	be	16, 50	kin	1	14, 20	27	20, 40	kur	16, 30	be
[kúr]	29	5, 51	be	10 uš	du$_6$	30	7, 30	28	16, 1[0]	kur	14	be
[kúr]	30	7, 52	be	14, 30	apin	1	11, 50	27	22, 10	kur	21	be
[kúr	30	10,] 10	be	19, 40	gan	1	17, 30	27	14, 50	kur	16, 10	be
[kúr	2]9	[6,] 53	be	13, 50	ab	30	13, 30	27	19, 30	kur	23, 20	be
kúr	30	10, 8	be	21, 50	zíz	1	21, 50	27	9, 40	kur	14	be
kúr	29	8, 2	be	17, 10	še	[30	18,] 20	27	11, 50	kur	19, 30	be
kúr	29	6, 4	be	11, 30	dir-[še	30	12,] 20	27	17, 50	kur	23, 50	be
kúr	30	10, 1	[b]e	2[0, 1]0	bar	1	[2]2, 50					

FIGURE 2c

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notice that in the first year there is a month VI_2, and at the end of the last year we have a month XII_2, indicating that here it has been necessary to insert an extra month in the normal 12-month lunar year in order to keep it in step with the solar year. Incidentally, these months are at this late date introduced according to a rigid pattern based on what I earlier called the Metonic cycle.

Each of the subsequent columns describes in an entirely numerical fashion some particular facet of the behaviour of the Sun or the Moon or both; thus the first preserved column (labelled I in figure 2*a*) gives the progress in degrees of the Sun, and, since we are dealing with conjunctions, also of the Moon (less 360°) in the ecliptic per month. The next column (II) gives the position of Sun and Moon at conjunction in the ecliptic. The first line tells that at the end of month XII of year 207 of the Seleucid Era the conjunction took place at 2° 2′6″20‴ of the sign Aries (a zodiacal sign means, by this time, simply a 30° section of the ecliptic). The next line says that the position of the subsequent conjunction of Sun and Moon was at 0° 52′45″38‴ of the sign Taurus. You get to the second position from the first by adding the number in line 2 of column I, thus, writing in the now standard sexagesimal notation:

column II, l. 1: Aries	2; 2, 6,20	
column I, l. 2: +	28;50,39,18	
column II, l. 2: Aries	30;52,45,38	= Taurus 0;52,45,38

and so on.

To dispel any notion that these positions are the results of incredibly accurate observations (to sixtieths of seconds, or thirds, of arc) I need only point out that the Moon is invisible for quite an interval before and after conjunction with the Sun, so that there is nothing to observe unless there happens to be a solar eclipse at that conjunction.

But even if the phenomenon had been observable – we have completely analogous texts for full moons which can be seen very well – the structure of column I, the difference column, completely rules out observations. The entries, you will note, decrease regularly by 18 in the second, i.e. the minutes' place, until a minimum is passed between lines 4 and 5. From line 5 the values increase, again by 18, until a maximum is passed between lines 10 and 11, whence they begin to decrease, and so on.

One gets from an ascending to a descending branch of this kind of function by following a very simple rule: if the application of the line by line difference d (here 0;18,0,0, to express it with the number of digits of the column) lead to a value larger than a certain fixed maximal value M, then the excess over M is subtracted from M to yield the next value of the function, and symmetrically about the minimum m. For column I we find

$$M = 30;\ 1,59,\ 0$$
$$m = 28;10,39,40$$

so that the reflexion in the maximum between lines 10 and 11 is executed thus:

column I, l. 10:	29;54,40, 2
$+d$:	0;18, 0, 0
	30;12,40, 2
$-M$:	30; 1,59, 0
	0;10,41, 2
which subtracted from M gives	29;51,17,58

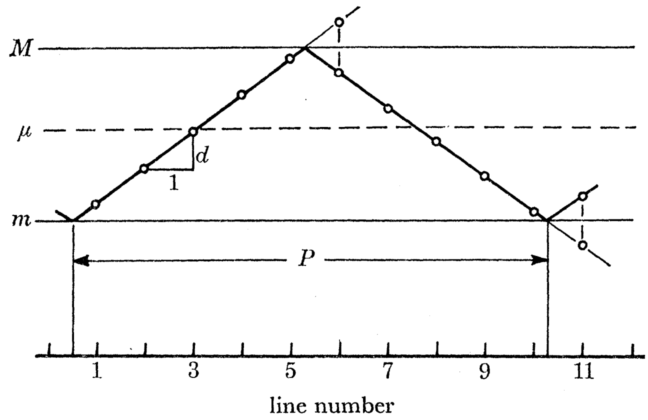

FIGURE 3

and this in indeed the value in column I, line 11. The general character of this function – we call it a zigzag function – is displayed in figure 3. This type of function, sometimes refined in various ingenious ways, is one of the two basic modes of describing a simply periodic component of a more complex astronomical phenomenon. Zigzag functions are easy to compute, and they have a simply controlled period

$$P = \frac{2(M-m)}{d}.$$

It must be emphasized that functions such as this are certainly not based on an underlying geometrical model; they are entirely arithmetical in character as are all the elements out of which the Babylonian astronomical systems are built.

I cannot in the time available attempt even a superficial analysis of the entire table; I can only try to give you an impression of the complexity of the underlying theories, and the depth of penetration into the behaviour of Sun and Moon, by describing summarily the character of some of the seventeen preserved columns of the text. I shall denote them both by a Roman numeral where the first preserved column receives no. I, as well as by a letter which, in the consistent terminology of Neugebauer (1955) characterizes the type of function contained in the column.

Column II (B) denotes, as said, the common longitude of Sun and Moon at conjunction. Columns III and IV (C and D) give the length of daylight and half the duration of night, respectively, in time degrees. They are based on zigzag functions which have been abbreviated. Column V (Ψ'') is a kind of eclipse magnitude, depending on lunar latitude. It is a slightly modified zigzag function whose period is the draconitic month, and when it lies between certain limits, an eclipse – or, rather, in this case the possibility of a solar eclipse – is announced by the term 'bar' (see, for example, line 5). The terms 'sig' and 'nim' mean that the Moon has negative or positive latitude, respectively.

Column VI (F) gives the lunar velocity at the time of conjunction in degrees per day; it is a zigzag function whose underlying period is the anomalistic month, as it should be. The next column, VII (G), has the same period as F, i.e. the anomalistic month, but is rightly out of phase with F. It denotes the excess, in time degrees, over 29 days of the time from one conjunction to the next, taking into account only the variation of lunar velocity. I shall give the

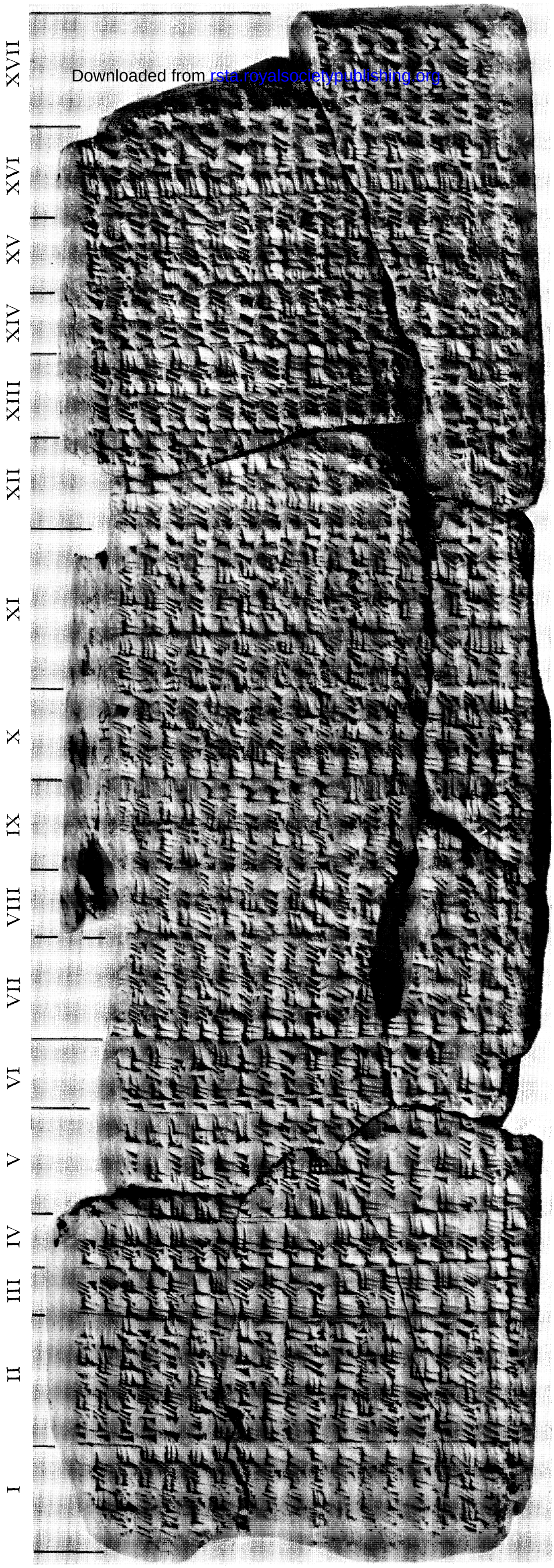

FIGURE 1. A.C.T. no. 122, reverse, $\frac{3}{4}$ size.

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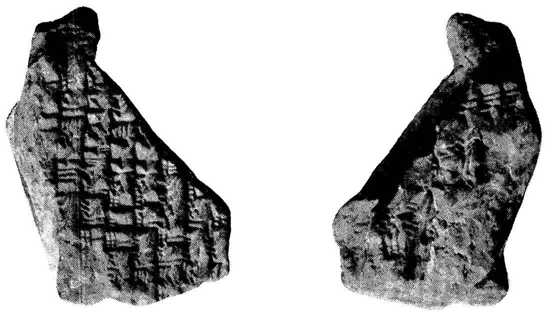

obverse

reverse

Figure 4. A.C.T. no. 20, actual size.

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JN		T	Φ	B		C	E	F	G	J		C′	K	M				P$_1$		
													I	II				III		
1069	obv.	XII	2, 13, 34, 15, 33, 20	6; 11, 15	♈	2, 57, 27, 30	+ 44, 44, 42	15, 3, 36, 33, 45	2, 40	18, 38, 25, 30	lal	−9, 22, 9	2, 12	še				bar		
1070		2, 47* I	2, 10, 48, 20	4; 18, 45	♉	3, 16, 12, 30	−2, 48, 23, 21	14, 21, 36, 33, 45	2, 53, 18, 45	57, 3,45	lal	−9, 22, 30	1, 46, 53	bar				gu$_4$		
1071		II	2, 8, 2, 24, 26, 40	2; 26, 15	♊	3, 28, 58, 30	−4, 47, 9, 3	13, 39, 36, 33, 45	3, 17, 50, 51, 51, 6, 40	57, 3, 45	lal	−6, 23	2, 14, 24	gu$_4$				sig		
1072		III	2, 5, 16, 28, 53, 20	0; 33, 45	♋	3, 34, 44, 30	−6, 45, 54, 45	12, 57, 36, 33, 45	3, 43, 39, 30, 22, 13, 20	57, 3, 45	lal	−2, 53	2, 43, 43	sig		3, 34, 5	šú	šu	1	
1073		IV	2, 2, 30, 33, 20	28; 41, 15	♋	3, 33, 30, 30	−5, 39, 19, 33	12, 15, 36, 33, 45	4, 9, 28, 8, 53, 20	57, 3, 45	lal	+ 37	3, 13, 2	šu	28	., 21, 3	šú	izi	1	15, 30
1074		V	1, 59, 44, 37, 46, 40	26; 48, 45	♌	3, 25, 16, 30	−3, 40, 33, 51	11, 33, 36, 33, 45	4, 35, 16, 47, 24, 26, 40	57, 3, 45	lal	+4, 7	3, 42, 20	izi	28	2, 38, 43	šú	kin	1	15, 30
1075		VI	1, 58, 37, 13, 20	25; 44	♍	3, 9, 30, 40	− 53, 14, 18	11, 16, 32, 48, 45	4, 56	32, 50, 33, 30	lal	+7, 52, 55	4, 31, 2	kin	28	4, 7, 41	šú	du$_6$	30	
1076		VII	2, 1, 23, 8, 53, 20	25; 44	♎	2, 49, 30, 40	+2, 51, 38, 33	11, 58, 32, 48, 45	4, 43			+10	4, 53	du$_6$	29	5, 14, 41	šú	apin	1	
1077		VIII	2, 4, 9, 4, 26, 40	25; 44	♏	2, 33, 42, 24	+4, 57, 54, 15	12, 40, 32, 48, 45	4, 17, 11, 21, 28, 53, 20			+7, 54, 8	4, 25, 5	apin	28	., 49, 36	šú	gan	30	
1078		IX	2, 6, 55	25; 44	♐	2, 25, 54, 8	+7, 4, 9, 57	13, 22, 32, 48, 45	3, 51, 22, 42, 57, 46, 40			+3, 54, 8	3, 55, 16	gan	29	2, 54, 20	šú	ab	1	
1079		X	2, 9, 40, 55, 33, 20	25; 44	♑	2, 26, 5, 52	+5, 13, 34, 21	14, 4, 32, 48, 45	3, 25, 34, 4, 26, 40			− 5, 52	3, 25, 29	ab	29	5, 28, 51	šú	zíz		
1080		XI	2, 12, 26, 51, 6, 40	25; 44	♒	2, 34, 17, 36	+3, 7, 18, 39	14, 46, 32, 48, 45	2, 59, 45, 25, 55, 33, 20			−4, 5, 52	2, 55, 40	zíz		2, 33, 11	šú	še		
1081		XII	2, 15, 12, 46, 40	25; 44	♓	2, 50, 29, 20	− 21, 54, 6	15, 28, 32, 48, 45	2, 40, 52, 57, 46, 40			−8, 5, 52	2, 32, 47	še				dir-še		
1082		XII$_2$	2, 16, 10, 55, 33, 20	23; 56, 15	♈	3, 9, 17, 30	−3, 22, 1, 45	15, 43, 15, 56, 15	2, 40	54, 38, 34, 30	lal	−9, 24, 5	1, 35, 58	dir-še				bar		
4193	rev.	2, 47* I	2, 2, 41, 28, 53, 20	20; 15	♎	3, 6, 50	+1, 14, 1	12, 18, 22, 30	4, 30, 48, 53, 20	47, 10, 18	lal	−9, 29	3, 34, 9	bar						
4194		II	2, 5, 27, 24, 26, 40	18; 22, 30	♏	3, 23, 21	+3, 47, 46, 12	13, 0, 22, 30	4, 5, 0, 14, 48, 53, 20	57, 3, 45	lal	−8, 15, 30	2, 49, 41	gu$_4$						
4195		III	2, 8, 13, 20	16; 30	♐	3, 32, 52	+5, 46, 31, 54	13, 42, 22, 30	3, 39, 11, 36, 17, 46, 40	57, 3, 45	lal	−4, 45, 30	2, 37, 22	sig						
4196		IV	2, 10, 59, 15, 33, 20	14; 37, 30	♑	3, 35, 23	+6, 38, 42, 24	14, 24, 22, 30	3, 13, 22, 57, 46, 40	57, 3, 45	lal	−1, 15, 30	2, 15, 4	šu						
4197		V	2, 13, 45, 11, 6, 40	12; 45	♒	3, 30, 54	+4, 39, 56, 42	15, 6, 22, 30	2, 48, 17, 36, 17, 46, 40	57, 3, 45	lal	+2, 14, 30	1, 53, 28	izi						
4198		VI	2, 16, 31, 6, 40	10; 52, 30	♓	3, 19, 25	+2, 41, 11	15, 48, 22, 30	2, 40	57, 3, 45	lal	+5, 44, 30	1, 48, 41	kin						
4199		VII	2, 14, 52, 35, 33, 20	10; 44	♈	2, 59, 30, 40	−1, 13, 1, 24	15, 23, 26, 15	2, 40	4, 18, 41	lal	+9, 57, 10	2, 45, 39	du$_6$				destroyed		
4200		VIII	2, 12, 6, 40	10; 44	♉	2, 39, 42, 24	−3, 54, 46, 24	14, 41, 26, 15	2, 43, 8, 53, 20			+9, 54, 8	2, 53, 2	apin	1					
4201		IX	2, 9, 20, 44, 26, 40	10; 44	♊	2, 27, 54, 8	−6, 1, 2, 6	13, 59, 26, 15	3, 5, 39, 45, 11, 6, 40			+5, 54, 8	3, 11, 33	gan	30	12				
4202		X	2, 6, 34, 48, 53, 20	10; 44	♋	2, 24, 5, 52	−6, 16, 42, 12	13, 17, 26, 15	3, 31, 28, 23, 42, 13, 20			+1, 54, 8	3, 33, 22	ab	1	13				
4203		XI	2, 3, 48, 53, 20	10; 44	♌	2, 28, 17, 36	−4, 10, 26, 30	12, 35, 26, 15	3, 57, 17, 2, 13, 20			−2, 5, 52	3, 55, 12	zíz		13				
4204		XII	2, 1, 2, 57, 46, 40	10; 44	♍	2, 40, 29, 20	−1, 44, 21, 36	11, 53, 26, 15	4, 23, 5, 40, 44, 26, 40			−6, 5, 52	4, 17	še						
4205		XII$_2$	1, 58, 17, 2, 13, 20	9; 52, 30	♎	2, 59, 55	+2, 21, 17, 48	11, 11, 26, 15	4, 48, 40, 14, 48, 53, 20	26, 7, 19	lal	−9, 42, 50	4, 12, 50	dir-še						
4206		2, 48 I	2, 0, 4, 48, 53, 20	8	♏	3, 18, 40	+4, 21, 24, 36	11, 38, 43, 7, 30	4, 53, 20	57, 3, 45	lal	−9, 22, 30	3, 46, 54	bar						

FIGURE 5

parameters of this zigzag function since they will be of importance in the following discussion; they are

maximum	$M =$	4,29;27, 5°
minimum	$m =$	1,52;34,35°
monthly difference	$d =$	22;30, 0°/m

$$\text{mean value } \mu = \tfrac{1}{2}(M+m) = 3,11;0,50° = 0;31,50,8,20 \text{ day}$$

to convert the time degrees into days (1 day = 6,0°). Thus column G implies that in the mean the Babylonian value of one synodic month is equal to

$$29 \text{ days} + \mu = 29;31,50,8,20 \text{ days.}$$

Column VIII (H) contains the differences of column IX (J). The values of J are a further correction to the length of the interval between conjunctions, this time depending on solar anomaly; thus we have that the length of the true synodic month is 29 days + G + J (since J's mean value is 0, the mean synodic month remains 29 days + μ). The values of G + J are listed in column X (K) which is the difference column of column XI (L).

Column L gives us the time of the conjunctions. Line 1 tells that the conjunction in month XII of the year 208 of the Seleucid Era took place on the 29th day at 1,2;43,50 time degrees after midnight (i.e. at about 4h11min a.m.). The hour in the next line is simply this augmented by the value in line 2 of the preceding column, and so on to the end of the text. To say what the date is, one must know whether the previous month had 29 or 30 days, the only two possibilities in a lunar calendar; this information is found in column XV (P_1). To give you a sense of the quality of the text, I shall compare the first and the last lines with the result of modern computations, according to which the conjunction in obverse, line 1 took place on 104 B.C., 23 March at 3h23min (text has about 4h11min) and that in reverse, line 19 on 101 B.C., 18 April at 0h45min (text has about 2h22min). Throughout the entire text the difference between ancient and modern computed values remains in the interval $1\frac{1}{2}$h ± 1h, where part of the 1.5h is accounted for by the deviation of the initial value.†

Let me just mention one more column before abandoning the text, namely, column XV (P_1), which concerns the first visibility of the new crescent which marks the beginning of a new month. Thus line 1 tells us that month I (bar) began on the 31 of month XII (hence the 1), and that the time from sunset to moonset was 15;40 time degrees, i.e. about 1h3min. These are precisely the two pieces of information given about the new Moon in the Astronomical Diaries which contain the observational material upon which the arithmetical theories were constructed (see Sachs, this volume, p. 43). Indeed, one of the principal aims of these theories is to generate forecasts of the astronomical events recorded in these diaries, and this column is an example.

By analysing texts such as this, and from another class of texts called procedure texts – they contain rules for computing the various columns of ephemerides – we have gained control over the theories underlying the procedures. To emphasize this, and also that these astronomical texts are entirely computed without the occasional injection of observations except possibly as ultimate initial values, I shall show you another text. Figure 4, plate 2, is a photograph of obverse and reverse of a fragment of a clay tablet, and it is, unfortunately, more typical of our material than the other.

† The modern values for the moments of conjunction are taken from a set of tables of all syzygies from 1000 B.C. to A.D. 1651 computed for the meridian of Babylon by Herman H. Goldstine. The tables will be published shortly by the American Philosophical Society. Dr Goldstine kindly put a copy of the original computer print-out at my disposal.

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Figure 5 shows our reconstruction of the text, first published by Neugebauer (1955) as A.C.T. No. 20, from which the fragment came (the preserved surface is outlined with dotted lines). I assure you that this reconstruction is quite secure though it admittedly is a *tour de force* which would not have been attempted without the aid of an electronic computer.

This text is also a lunar ephemeris, but here the obverse concerns new moons, while the reverse treats of full moons, both for the year 167 of the Seleucid Era (145/4 B.C.). Both the previous text and this come from Babylon and both are entirely arithmetical in structure, but they are computed according to two quite distinct systems: the former belongs to system B, this to system A, to use the now standard terminology for the two major Babylonian astronomical systems.

A characteristic difference between the two systems – though far from the only one – lies in the manner of computing longitudes. As I have already mentioned, the monthly progress in the ecliptic of a syzygy is determined by a zigzag function in system B. In system A the approach is quite different. Here the monthly progress of the syzygy in longitude, $\Delta\lambda$, is, to be sure, not explicitly displayed as in column A of system B, but a glance at the longitude column (B) of our text should suffice to reveal that the longitudes fall in groups, within which $\Delta\lambda$ alternately assumes the value 30° (i.e. the syzygy advances precisely one full zodiacal sign per month), and the value 28;7,30°. In my restoration of the text in figure 5, the groups are separated by dotted lines. The transitional values of $\Delta\lambda$ across these dotted lines lie between 30° and 28;7,30°.

The underlying scheme may be described in terms of the following model for the motion of the Sun (it happens that the lunar anomaly has so small an effect on the *position* of syzygies that it may be ignored in first approximation): the Sun's velocity is a piecewise constant function of solar longitude, namely,

from Virgo 13° to Pisces 27°: V = 30°/month,
from Pisces 27° to Virgo 13°: v = 28;7,30°/month.

The monthly progress $\Delta\lambda$ of the Sun, and hence of the syzygy, is then mostly either V or v, but it assumes an intermediary value, which is readily computed, when a boundary between the two zones is transgressed in the course of a month. Figure 6 shows the character of this step function (heavy horizontal lines); the values $\Delta\lambda$ agree mostly with this generating or velocity function except in intervals of length V and v preceding the discontinuities. Here $\Delta\lambda$ is drawn as a lighter skew line segment, for $\Delta\lambda$ is a piecewise linear, continuous function of λ as can easily be shown. Such step functions, together with a rule for deriving progress in longitude from them, is the alternative Babylonian arithmetical device for describing periodic phenomena. It seems, at first, cruder than the zigzag functions but is actually much more flexible.

My description of the system A scheme for finding longitudes of consecutive syzygies is frankly in the modern kinematical idiom, using a notion like the instantaneous velocity of the Sun; I shall presently give an explanation of a system A model which avoids such anachronisms, but before leaving our text I should like to identify its columns briefly.

Column T gives year (in the Seleucid Era) and month and column Φ is a function in phase with lunar velocity and the basis of finding the later column G. Column B, as just said, lists on the obverse the common longitude of Sun and Moon at conjunction, and on the reverse the longitude of the Sun at midmonth increased by 180°, which is the longitude of the full Moon. Column C gives the length of daylight and column E is actually lunar latitude (in units

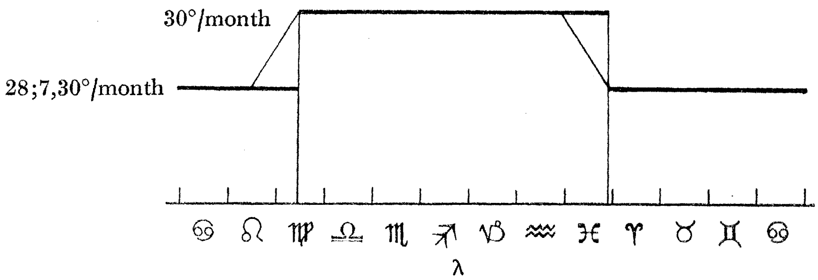

FIGURE 6

'barleycorns', 72 of which equal one degree; the integral barleycorns occupy the first two sexagesimal places). Lunar latitude turns out to be a very simple function – a slightly modified zigzag function – of the Moon's elongation from the ascending node; this nodal elongation is easily found from column B giving lunar longitude combined with the underlying assumption that the node retrogresses by the constant amount 1;33,55,30° per month, and some initial position of the node (v. d. Waerden 1966). Again, eclipse warnings would be issued whenever the new or full moon has smallest latitude at a nodal crossing; this is done with an eclipse magnitude function depending simply on column E, but which I have not bothered to reconstruct.

Columns G and J together give, as before, the excess over 29 days of the time from conjunction to conjunction, or opposition to opposition, where G depends on lunar, and J on solar anomaly. Their structures are more complicated than those of their counterparts in system B, and I shall pass them by except for pointing out that J's mean value here is negative (for details see Aaboe 1971). Column C′ gives a correction due to the variation in length of daylight, column K the sum of G, J and C′, and column M lists the moment of syzygy (on the obverse date and time degrees of conjunction *before* sunset of the day), and the moments proceed from line to line by the amount K. Finally, column P, preserved on obverse only, gives information about the visibility of the new crescent; '1' means that the previous month turned out to be full (30 days long), '30' that it was hollow (29 days long), and the following numbers give the computed time from sunset to moonset.

I realize well that this cursory presentation of two lunar texts is unsatisfactory and, in particular, that it fails to show the beautiful, simple, yet highly sophisticated manner of treating technical details. I hope, however, to have conveyed some sense of the complexity of Babylonian lunar theory, and that the Babylonian theoretical astronomers had succeeded in isolating the essential periods of lunar and solar motion and in putting them to proper use.

My last example from Babylonian astronomy is not a text, but rather the result of an analysis of a planetary model belonging to system A. The scheme is for the longitude of Mars at one of its characteristic synodic phenomena, say, at first stationary point, and I have chosen it for several reasons: to display the role of an unexpected branch of mathematics in such schemes, to emphasize the flexibility of the approach to astronomy of system A, and finally, and most importantly, to point to a possible connexion between the sort of observations we know the Babylonians to have recorded and their theoretical constructs.

The first, and to us unfamiliar, feature of Babylonian planetary theory lies in the very question one asks.

Since Ptolemy's *Almagest* we have wanted our planetary theories to enable us to answer the question: given the time, where is the planet? Thus we consider *time* the independent variable and seek means of deriving all other information from it.

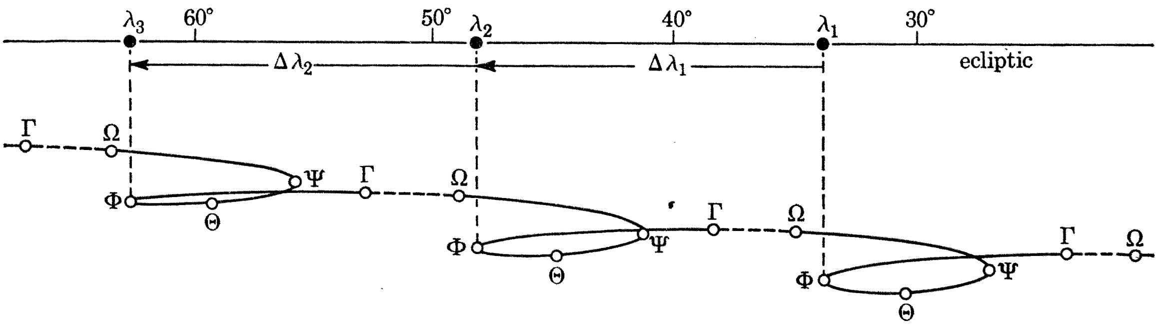

FIGURE 7

The Babylonian approach is entirely different. First, almost all interest, at least primarily, is focused upon the planet only when it is in one of its characteristic synodic situations: for an outer planet they are – see figure 7, which represents a run of Saturn with latitude exaggerated four times – (i) first appearance (Γ), (ii) first stationary point (Φ), (iii) opposition (Θ), (iv) second stationary point (Ψ), (v) last appearance (Ω) (the capital Greek letters are the now standard manner of referring to these synodic phenomena).

The next bold simplification is that we disregard all but one of these, say the first stationary point Φ, and we now ask the question: if we are given the longitude and the time at which a certain planet happens to be at a first stationary point, where and when will it next be at a first stationary point? What we, in the Babylonian mode, consider and wish to reproduce is then, a sequence of discrete points, in time and longitude.

Figure 8 is a graphical representation of the arithmetical model upon which the system A

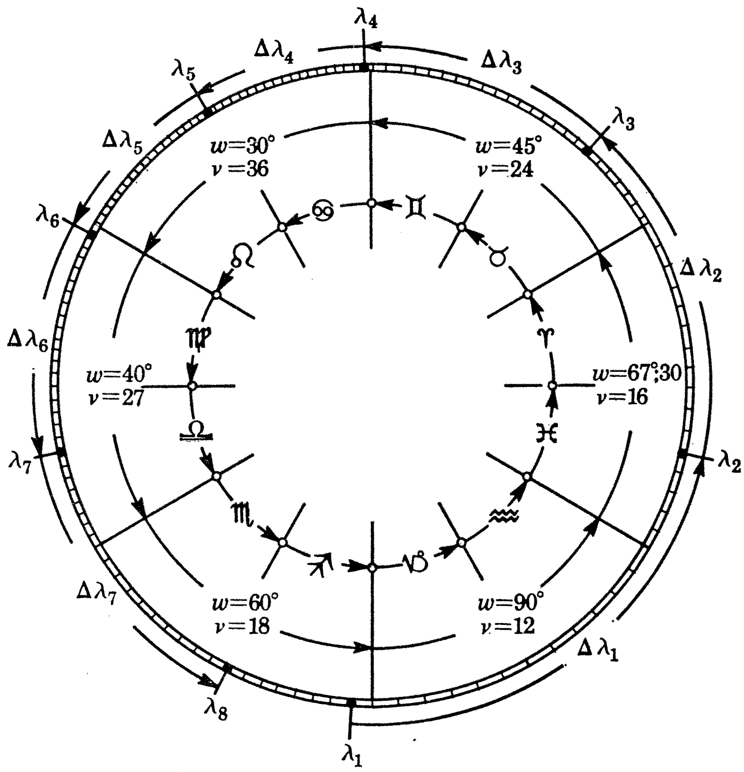

FIGURE 8. Mars, system A: 133 synodic phenomena ~ 18 revolutions ~ 2.133+18 years.
1 synodic arc = 18 intervals.

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scheme for Mars is based (Aaboe 1964). The ecliptic is divided into six arcs each consisting of precisely two zodiacal signs, thus:

α_1: Pisces and Aries,	α_4: Virgo and Libra,
α_2: Taurus and Gemini,	α_5: Scorpio and Sagittarius,
α_3: Cancer and Leo,	α_6: Capricornus and Aquarius.

Further, each of these arcs is divided into a certain number (ν) of intervals, thus α_1 into $\nu = 16$ intervals, α_2 into 24, and so on, as indicated on figure 8. The total number of intervals is 133 which, as we shall see, is an astronomically significant number for Mars.

The rule for finding the longitude of Mars at successive first stationary points (Φ) is the following: let the longitude of the initial stationary point be represented by the black dot near the end of Sagittarius at the bottom of figure 8 (it is, actually, 26° 40′ of Sagittarius); we now progress in steps consisting always of 18 intervals each, whatever the length of the intervals. Thus the second Φ for Mars will be near the middle of Pisces at the end of the step, *synodic arc* is the proper term, marked $\Delta\lambda_1$; the third in Taurus at the end of $\Delta\lambda_2$, etc., and the eight Φ, the last indicated on the figure will be at the beginning of Sagittarius (actually at Sagittarius 3° 20′), which has brought the phenomenon almost once around the ecliptic.

The length of a synodic arc depends only on where in the ecliptic it begins, i.e. on the lengths of the 18 small intervals that constitute it, and it varies considerably (the values w, which are 18 times the length of the intervals, i.e. the length of the synodic arc if a new zone were not encountered, are the parameters given in the Babylonian texts, and they play the same role as the two values of solar velocity mentioned above).

It should be clear that after 133 synodic arcs, but not earlier, the phenomenon will return precisely to its point of departure, for the total number of intervals covered will be 133 times 18 which, since the ecliptic is divided into 133 intervals, correspond to 18 complete revolutions. It is only the synodic phenomenon that leaps by $\Delta\lambda$ or 18 intervals; Mars itself, being a swift planet, moves in its actual motion one additional revolution between two synodic phenomena of the same kind, and the Sun revolves twice in the ecliptic before it catches up with Mars after a further $\Delta\lambda$ (a synodic phenomenon takes place when the planet and the Sun have a certain fixed relation). Thus the 133 synodic arcs, which constitute 18 complete revolutions correspond also to

$$133+18 = 151 \text{ revolutions of Mars}$$

in the ecliptic and

$$2\times 133+18 = 284 \text{ revolutions of the Sun}$$

or, with another word, years. Now the time span of 284 years is an excellent period of Mars, a refinement of the sort of periods I discussed earlier. Thus, this system A scheme, as all similar schemes, is built upon a foundation of a sound period relation which assures that the errors inherent in this, as in all, theoretical approximations to natural phenomena do not accumulate arbitrarily.

I shall not touch on the scheme for finding the corresponding moments, beyond stating that the time interval from one Φ to the next is simply

$$\Delta t = \Delta\lambda + C,$$

where C is a suitable constant, and that this works very well (Aaboe 1958).

Several comments are in order. First I must warn you most emphatically not to be misled by my graphic representation of the scheme into thinking that there may have been some sort of geometrical model underlying the Babylonian procedures. This is not so; all the Babylonian astronomical models, as I still feel justified in calling them, were entirely arithmetical in character. It is not very difficult to give the essence of my description of the Mars model completely in arithmetical or perhaps number-theoretical language.

Secondly, except for the choice of initial values, the independent variable is here not time, but the longitude of a synodic phenomenon. It is that which entirely determines the length of the step forward, in longitude as in time. Similar, indeed mostly identical, models are applied to the other synodic phenomena. If, finally, one wishes to know where the planet in question is at a given arbitrary moment, this is determined by interpolation between the appropriate synodic phenomena by means of various interpolation schemes which may be of as much as third order (Neugebauer 1955; Huber 1957).

Lastly, I should like to point to a possible way of deriving theoretical schemes of this sort from the kind of observations we know the Babylonians to have made and recorded. Professor Sachs will describe the texts he calls Astronomical Diaries which survive in large numbers, all deriving from unscientific excavations of what must have been an astronomical archive somewhere in the ruins of Babylon. These texts date roughly from the last seven centuries B.C. They are unique as a corpus of ancient historical documents, but what is of importance in this context is that they contain observations of precisely the sort of phenomena reproduced by the Babylonian theoretical schemes. These observations are at first sight disappointing, since, for the phenomena in which we are interested, they are of the form: *In year n of King N, month x, day y, Mars reached its first stationary point; it was in the zodiacal sign Z.* This means that the longitude of the phenomenon which, as we have seen, is at the basis of the theory, is given only as a zodiacal sign (except in unusual circumstances), i.e. the planet is fixed only within an interval of length 30°.

This, then, is the problem, if not dilemma, which has bothered students of Babylonian astronomy for a number of years: how can one from observations of such crudity derive such excellent schemes?

The answer I shall suggest presupposes first, that one had arrived at a good period relation like

$$133 \text{ synodic phenomena} \sim 18 \text{ revolutions of the phenomenon}$$
$$\text{in the ecliptic} \sim 151 \text{ revolutions of Mars} \sim 284 \text{ years,}$$

to keep to Mars as our example. May I here insert the remark that one need not observe Mars for a full 284 years to reach these relations; it is very possible to construct such periods from much shorter ones by correcting for their deficiencies, but I cannot go into details with this in this brief time.

Thus one knows, to begin with, that there are to be 133 intervals in all, and that there must be 18 of them from phenomenon to phenomenon. The model is determined as soon as these 133 intervals are distributed properly on the ecliptic, so the aim is to fix the values v, whose sum is already known.

To that end one may now take from the observational records a run of observations of consecutive synodic phenomena of a certain kind, preferably for some reasonable subperiod, and sort them out according to zodiacal sign; i.e. one keeps a tally, sign by sign, of the phenomena. It appears very soon that certain signs, particularly Cancer and Leo, are very popular, while others, Capricorn and Aquarius, are not. First, this is an indication that longitude is a sensible choice of independent variable and, second, one gets from such a tally a feeling for the proportionate distribution of the intervals in the several signs.

Not everything is explained by such a tally; thus, the decision to consider six zones of length precisely two signs each is certainly not the only possible one. Further, it is necessary that the values ν be nice numbers – nice in the sense that their reciprocals have finite sexagesimal expansions – in order that the number-theoretical properties be strictly obeyed by the derived ephemerides. But granting this choice and these limitations, it turns out that one really has very little freedom left for distributing the intervals. A comparison between modern planetary tables and ancient ephemerides based on such models shows excellent agreement even in the case of as difficult a planet as Mars.

I shall finish my discussion of Babylonian mathematical astronomy with a few remarks of a more general nature. First, I wish to emphasize the lateness of the relevant texts. The vast majority of them belong to the Seleucid period, i.e. roughly the last three centuries B.C., and though it is impossible to point to an earliest date, I feel reasonably sure that all the presently known and understood texts, some 400 in number, come from the last five centuries B.C., to give a very generous earlier limit. By the beginning of the Seleucid Era the theories were certainly fully developed, and our texts continue to the very end of cuneiform literacy. The creation of mathematical astronomy is thus one of the last, as well as one of the finest, original efforts of Mesopotamian culture, an event without precedent anywhere, and with great consequences.

Secondly, I want to stress that the creators of these astronomical theories, whoever they were, were able to draw from, and combine, a peculiar set of available ingredients which we know of from other textual material. Principal among them are, first, a particular kind of mathematics and, secondly, a body of continuously recorded observations

As to the former the mathematical cuneiform texts fall into two groups in respect of chronology: one Old-Babylonian from the beginning of the second millennium B.C. and the other Seleucid (Neugebauer & Sachs 1945). There is, however, hardly any difference between these two groups of documents in content or character. At the basis of Babylonian mathematics is the sexagesimal number system which you have seen amply in use in the astronomical tables above, a system which reduces the four basic arithmetical operations to trivialities. Though much of what we would call geometrical knowledge is incorporated in Babylonian mathematics – foremost among it a command of the Pythagorean theorem, as we are wont to call it, though it antedates Pythagoras by a millennium – its chief concern is still with numerical problems, algebra, and perhaps number theory, even when the problems at first sight may seem to wear geometrical garb. It is, then, not surprising to find an *arithmetical* treatment of astronomical phenomena into which, further, period relations are built through simple number-theoretical devices.

As to the second, the recorded observations are embodied principally in the Astronomical Diaries as I have already mentioned; here I shall just add that there is little doubt that the Babylonian astronomers had available to them an archive of sustained observations going back to about 700 B.C. The existence of a connexion between the observational and theoretical material is clear, for the goal of the theories is, in the large, to reproduce and predict precisely the kind of phenomena and quantities recorded in the Diaries.

It is idle to ask if the combination of this kind of mathematics and observations was necessary for the creation of mathematical astronomy. The historical fact is that it was sufficient, and that wherever else we encounter scientific mathematical astronomy we can detect, directly or indirectly, the influence of the Babylonian forerunner.

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I shall try to justify this, necessarily very briefly, and first I shall point out a passage from the beginning of *Almagest* IV (written about A.D. 150) in which Ptolemy is concerned with establishing relations between the various periods of solar and lunar motion. He says that Hipparchus (about 150 B.C.) had shown that the smallest interval between two eclipses which produces repetition in solar and lunar anomaly is

126007 days 1 h = 4267 synodic months = 4573 anomalistic months = 4612 rotations of the Moon in long. less $7\frac{1}{2}°$.

Hence it is found by division, says Ptolemy, that

1 synodic month = 29;31,50,8,20 days.

The difficulty with this is that if we do divide the time interval, in days and hours, by the number of synodic months, we do not get 29;31,50,8,**20** but 29;31,50,8,**9**,... days as the length of the mean synodic month. (This discrepacy has been noted repeatedly in the astronomical literature, e.g. by al-Bīrūnī and by Copernicus.)

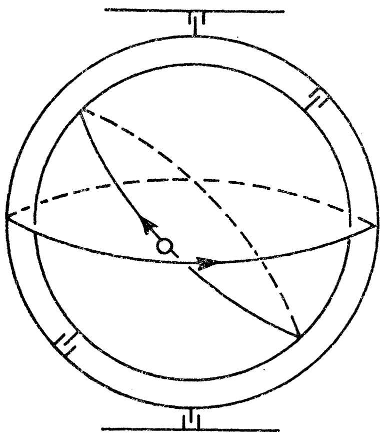

FIGURE 9

Now 29;31,50,8,20 days is precisely the value of the synodic month used in Babylonian lunar system B texts, as we saw above in the comments on column G (already Kugler was aware of this (Kugler 1900, p. 24)). There can, then, be no doubt whatever that Hipparchus got this parameter from Babylonia, and the rest of his relation can be shown to have Babylonian origins as well (Aaboe 1955).

All the works of Hipparchus are lost but one, a not too informative commentary on an astronomical poem by Aratus. Thus all we say about Hipparchus has essentially to be based upon secondary sources, principally Ptolemy who, however, is generous with references to his predecessor. It appears that what Hipparchus was engaged in was to adapt geometrical astronomical models of a certain kind to new purposes.

There was, of course, a tradition for such models in the Greek-speaking world. May I remind you first of the homocentric spheres of Eudoxos (*ca.* 370 B.C.), an aesthetically pleasing theory which was rescued from various secondary references and restored in a now classic paper by Schiaparelli (1925).

The basic device in this scheme for simulating planetary behaviour is two spheres, the inner being able to rotate relative to the outer, and the outer being able to rotate relative to some fixed frame; the two axes of rotation are inclined to each other, and the planet is affixed to the 'equator' of the inner sphere. This arrangement is schematically shown in figure 9. The two spheres are made to revolve in opposite directions, but equally swiftly. If the axes coincided, the planet would not move at all, for the two equal, but opposite rotations would cancel each other. But when the axes are inclined the planet will travel in a path shaped like a figure 8, as is shown in figure 10. Here only one sphere is drawn, but both axes and their equators are represented. The curve is called a hippopede, a horsefetter, and, as can be shown by elementary means, it happens to be the intersection between the sphere and a right cylinder as indicated in figure 10 (Neugebauer 1953, 1957).

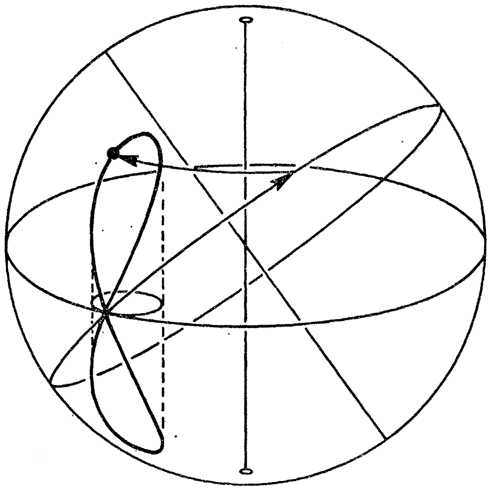

FIGURE 10

The apparatus is now placed so that the vertical line of symmetry of the figure 8 is the ecliptic, and the whole thing is given a forward motion in longitude (this may be achieved by yet another rotating sphere). Figure 11 shows what the path of the planet looks like, when observed from the common centre of the spheres, for different forward motions. At the top we have the hippopede itself, and as the forward motion increases we see a curve which indeed looks somewhat like the apparent path of a planet with its stationary points and retrogradations (cf. figure 7). If, however, the forward motion is too swift then the planet can no longer manage to become retrograde, but is merely slowed down, as shown in the two lowest graphs.

When we toy like this with various velocities, we assume that we are free to assign them at will. But when we deal with a specific planet, that is not so, for the period of the motion in the hippopede, i.e. of the rotations of the first two spheres, must be the planet's synodic period, and the forward motion in longitude must be the mean synodic arc $\Delta\lambda$ per synodic period. Both of these are determined within rather narrow bounds by even crude period relations of the sort I mentioned above. Our only really free choice in this model is, then, the inclination between the two axes in figure 9.

I have already mentioned the simple period relation for Venus:

5 synodic cycles = 8 years = 8 revolutions in the ecliptic of Venus.

If we, then, are to consider an Eudoxian model for Venus, we must let each of the two inner spheres revolve 5 times in 8 years, in opposite directions, to make Venus travel 5 times through the hippopede. Further, the hippopede must be carried 8 times around the ecliptic in the same span of 8 years. It can, however, be shown that this motion is so fast that no matter what inclination between the axes of the first two spheres we choose, we have a situation like that represented in the last graph in figure 11: Venus just cannot become retrograde. Precisely the same holds for Mars.

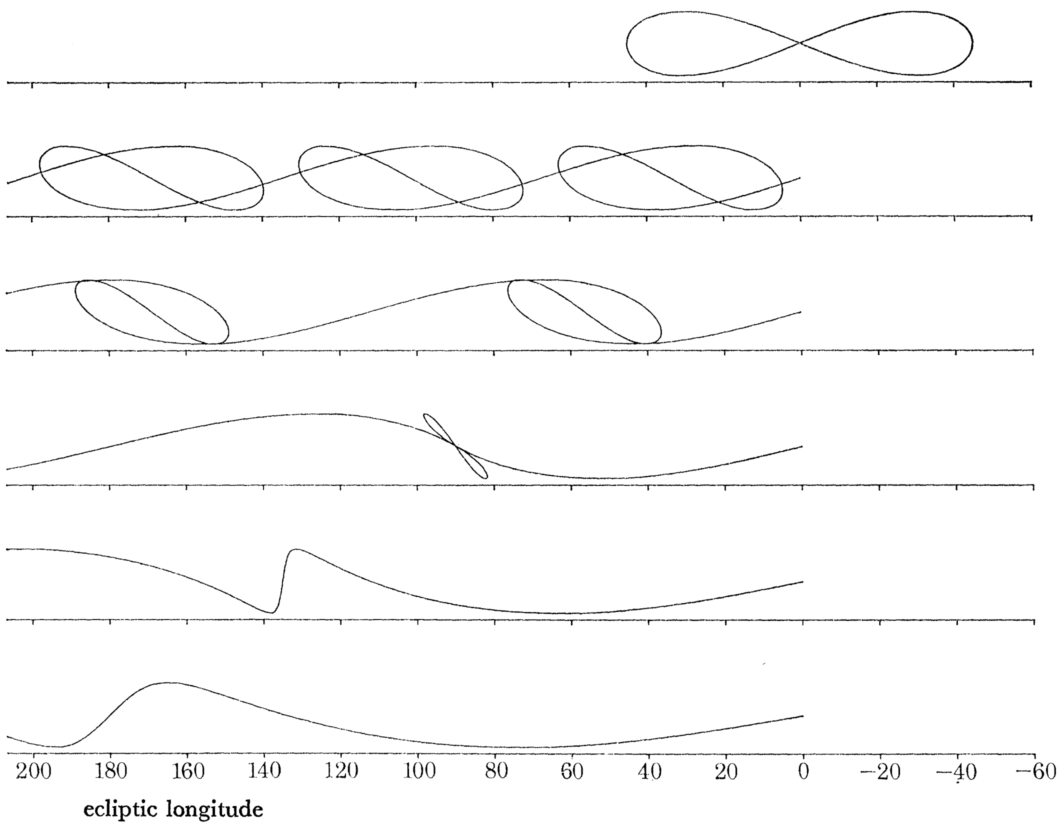

FIGURE 11

I wish to emphasize this point strongly because I believe it reveals that the purpose of these models was to serve as a *qualitative* description of planetary motion. Indeed, it cannot be anything else, for if one wants to get a quantitative result out of such a model one has little latitude in the choice of periods; thus, no matter how one manages to follow the resultant motion of the planets, the models for Venus and Mars will fail most drastically by their inability to produce retrograde motion, one of the most conspicuous phases of planetary behaviour, and the very phenomenon, I suspect, that the models were created to account for. There are other difficulties in putting these models to quantitative use, e.g. that all retrograde arcs for a planet that does become retrograde are of equal length, and that the mathematical techniques (spherical trigonometry) necessary for deriving numerical results from them were not created until some two centuries after Eudoxos.

May I further remind you of another type of geometrical planetary model, one which is much better known, for it became supremely successful in Ptolemy's refined version. In figure 12, the paper is the plane of the ecliptic viewed from the north, O the observer at the centre of a circle called the deferent, C a point travelling on the deferent and centre of a second circle called the epicycle upon which the planet P moves. As before, the periods of the motions are determined: P must move once around the epicycle *relative to the line* OC in one synodic period,

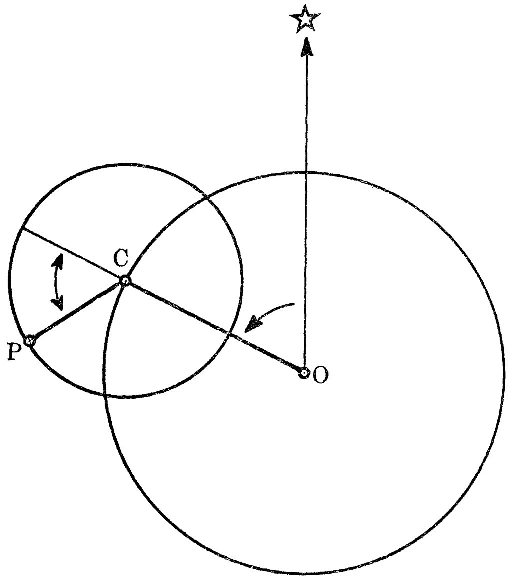

FIGURE 12

and C must move the synodic arc $\Delta\lambda$ per synodic period on the deferent relative to a line from O to, say, a fixed star. If we normalize and set the radius of the deferent OC = 1, there remains then to determine the epicyclic radius CP to suit the specific planet's requirements.

There is, however, a second, often ignored, free choice, and that is of the sense of rotation of the planet on the epicycle. We now take it as a matter of course that the planet has to move counterclockwise on the epicycle, for then the model corresponds closely to a correct first-order representation of geocentric planetary motion; but the contrary choice is in principle possible, for it, too, will make P behave like a planet, except that now it will become retrograde when farthest from O. There is evidence in a Greek papyrus that this choice was, in fact, made by some; this is of interest for a kinematical analysis shows that a model with P going clockwise, and otherwise given the proper periodic motions, is also incapable of making Venus and Mars retrogress, no matter what value is given to the epicyclic radius (Aaboe 1963).

Though our evidence of pre-Hipparchian astronomy is scant and fragmentary, I feel justified by such analyses in my belief that the purpose of early geometrical astronomical models was not to serve as a basis for determining positions of celestial bodies at a given time with some numerical accuracy, but that it rather was like that of the orreries: to display in a qualitative manner how the motions of celestial bodies may be generated.

We can now see more clearly what Hipparchus set out to do: he took one variety of qualitative models, epicyclic models with the right sense of rotation on the epicycle, refined them by employing eccentric deferents, determined reasonable parameters for these models, and made them yield numerical results through the application of trigonometry, a branch of mathematics which he probably created for the purpose. He was successful with models for the Sun, and for the Moon at syzygies, but failed to solve the planetary problem to his satisfaction. His work was completed by Ptolemy some three centuries later.

We saw that Hipparchus had at his disposal for this task important and refined astronomical Babylonian parameters. It is further beyond doubt that he also had access to Babylonian observational material going back to *ca.* 700 B.C. – in what form we do not, and probably shall not know – and so had even Ptolemy. Important though this is, I believe that the Babylonian influence on Hipparchus was even more fundamental. I am convinced, though I cannot offer proof, that whether or not he understood the technical details of the Babylonian ephemerides, it was from them that he got the idea of the possibility and desirability of a quantitative description of astronomical phenomena that could yield fine numerical predictions.

It is on these facts and on this belief that my case for the Babylonian origin of all endeavour in the exact sciences rests. This is a substantial claim. To make it seem more plausible may I try to trace hurriedly the development and the paths of transmission of astronomy from the time of Hipparchus.

We have very little evidence of astronomical activities in the Hellenistic world between Hipparchus and Ptolemy – it is largely due to the excellence of Ptolemy's work, I am sure, that so little primary evidence of pre-Ptolemaic Greek astronomy survives. It seems certain, however, that it was during this interval that astronomical models and parameters were transmitted to India. In the Hindu astronomical writings we find no trace at all of Ptolemy's refined models; the elements we recognize are, to be sure, first geometrical models of the Greek variety, but there is a large admixture of arithmetical schemes of obvious Babylonian origin. (Among them are some based precisely on the Mars model I discussed above in § 4, with its six two-sign zones, admittedly badly corrupted and probably useless, but still unmistakably recognizable (Neugebauer & Pingree 1970).)

Pingree (1963) maintains that, though direct contact between Mesopotamia and India cannot be ruled out, it is still overwhelmingly likely, on linguistic grounds among others, that these schemes reached India via Hellenistic Greece. This, incidentally, gives us indirect evidence of a wider knowledge of Babylonian techniques in the Greek-speaking world than is indicated by the fragile direct evidence.

The culmination of the Greek high tradition in astronomy is Ptolemy's *Almagest*, the importance and influence of which can hardly be overrated. Ptolemy's goals were those of Hipparchus – which, if I am right, were inspired by the Babylonian example – and he succeeded where Hipparchus gave up: in constructing a satisfactory planetary theory and a lunar model that worked at quadrature as well as at syzygy.

Islamic astronomy began with adaptations of Indian schemes, but when Ptolemy's *Almagest* became known and understood – not only do we have translations of it into Arabic, but though, e.g. al-Battānī's astronomical work follows the *Almagest* closely it is intelligently up-dated in essential places – it became the standard against which all advanced astronomical efforts were measured, even by those who disagreed with Ptolemy on various points, such as al-Ṭūsī, Quṭb al-Dīn, and ibn al-Shāṭir.

In the Latin West we find first translations of Arabic works based on Indian mixtures of Greek and Babylonian elements, later, versions in the Ptolemaic tradition, and finally proper translations of Ptolemy himself. Here again Ptolemy's methodology becomes the accepted standard, and so it remains until Kepler, and it is clear that mathematical astronomy was the principal motivation for the continued study of various branches of mathematics, among them trigonometry.

Thus the astronomical tradition in the West is linked to Babylonian astronomy. Mathematical astronomy was, however, not only the principal carrier and generator of certain mathematical techniques, but it became the model for the new exact sciences which learned from it their principal goal: to give a mathematical description of a particular class of natural phenomena

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capable of yielding numerical predictions that can be tested against observations. It is in this sense that I claim that Babylonian mathematical astronomy was the origin of all subsequent serious endeavour in the exact sciences.

The photographs of the cuneiform texts, figures 1 and 4, are published through the courtesy of the Trustees of the British Museum. Figures 2*a*, *b*, *c* are reproduced from the Astronomical Cuneiform texts with Professor Neugebauer's kind permission.

The reconstruction of A.C.T. no. 20 (figure 5) is published here for the first time. It was made possible by tables produced by the Yale computer according to programs by Miss Janice Henderson and by Mr Christopher Anagnostakis. This was done as part of a study of Babylonian astronomy supported by a grant from the National Science Foundation. Mr Anagnostakis also somehow caused the same computer to draw figures 10 and 11.

I wish to acknowledge my debt of gratitude to all.

References (Aaboe)

Aaboe, A. 1955 On the Babylonian origin of some Hipparchian parameters. *Centaurus* **4**, 122.
Aaboe, A. 1958 Babylonian planetary theories. *Centaurus* **5**, 209.
Aaboe, A. 1963 A Greek qualitative planetary model of the epicyclic variety. *Centaurus* **9**, 1.
Aaboe, A. 1964 On period relations in Babylonian astronomy. *Centaurus* **10**, 213.
Aaboe, A. 1971 Lunar and solar velocities and the length of lunation intervals in Babylonian astronomy. *K. danske Vidensk. Selsk.* (*Mat. -fys. Medd.*) **38**, 6.
Aaboe, A. 1972 Remarks on the theoretical treatment of eclipses in antiquity. *J. Hist. Astron.* **3**, 105.
Huber, P. 1957 Zur täglichen Bewegung des Jupiter nach babylonischen Texten. *Z. Assyriologie* N.F. **18**, 265.
Kugler, F. X. 1900 *Die Babylonische Mondrechnung*. Freiburg im Breisgau.
Neugebauer, O. & Sachs, A. 1945 *Mathematical Cuneiform texts*. New Haven (American Oriental Series vol. 29).
Neugebauer, O. 1953 On the 'Hippopede' of Eudoxos. *Scr. math.* **19**, 225.
Neugebauer, O. 1955 *Astronomical Cuneiform texts*. 3 Vols. London: Lund Humphries.
Neugebauer, O. 1957 *The exact sciences in antiquity*, 2nd ed. Providence: Brown University Press. Reprint by Dover Press 1968.
Neugebauer, O. & Pingree, D. 1970 The Pañcasiddhāntikā of Varāmihira. *K. danske Vidensk. Selsk.* (*Hist.-filos. Skr.*) **6**, **1** (2 parts).
Parker, R. A. & Dubberstein, W. H. 1956 *Babylonian Chronology 626* B.C.–A.D. *75*. Providence: Brown University Press.
Pingree, D. 1963 Astronomy and astrology in India and Iran. *Isis* **54**, 229.
Sachs, A. 1948 A classification of the Babylonian astronomical tablets of the Seleucid period. *J. Cuneiform Studies* **2**, 271.
Schiaparelli, G. 1925 *Scritti sulla storia della astronomia antica*. P.I -Scritti editi. vol. II. Bologna.
v. d. Waerden, B. L. 1966 *Anfänge der Astronomie*. Groningen.

Note added in proof, October 1973

On P.Mich. 149, the Greek papyrus with evidence of wrong choice of sense of rotation on the epicycle, see now also

Neugebauer, O. 1972 Planetary motion in P.Mich. 149. *Bull. Am. Soc. Papyrologists*, **9**, 19.

and on the relation between Babylonian and early Indian astronomy see further

Pingree, D. 1973 The Mesopotamian origin of early Indian astronomy. *J. Hist. Astron.* **4**, 1.

FIGURE 1. A.C.T. no. 122, reverse, $\frac{3}{4}$ size.

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bverse

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everse

FIGURE 4. A.C.T. no. 20, actual size.